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Quark Confinement and Number of Flavors in Strong Coupling Lattice QCD

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The problem of whether there is a constraint on the number of flavors for quark confinement in QCD is numerically investigated on a lattice with Wilson fermions as quarks. It is shown that even in the strong coupling limit, when the number of flavors exceeds 7, quarks are not confined and chiral symmetry is not spontaneously broken for light quarks.

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The fundamental properties of QCD are quark confinement, asymptotic freedom, and spontaneous breakdown of chiral symmetry. It is well known that if the number of flavors exceeds 17, asymptotic freedom is lost. Then the question which naturally arises is whether there is a constraint on the number of flavors for quark confinement and/or the spontaneous breakdown of chiral symmetry. Here we would like to investigate numerically quark confinement and chiral symmetry versus the number of flavors, taking the Wilson formalism [1] of fermions on the lattice for quarks, because this is the only known formalism which describes any number of flavors in terms of a local action. We use the same method as in a previous paper [2] to discriminate the phases of QCD with various numbers of flavors: With the quark mass defined through the axial-vector-current Ward identity, the pion mass at zero quark mass determines whether chiral symmetry is spontaneously broken or not. It will turn out that confinement is closely related with the spontaneous breakdown of chiral symmetry.

We generate gauge configurations using the hybrid-molecular-dynamics R algorithm [3] with the molecular dynamics time step $\Delta\tau = 0.01$, unless otherwise stated. The inversion of the quark matrix ($x = D^{-1}b$) is made by a minimal residual method or a conjugate gradient (CG) method. The lattice sizes are $8^2 \times 10 \times T$ ($T = 4$, 6, or 8) and $18^2 \times 24 \times T$ ($T = 18$). When the hadron spectrum is calculated in the former case, the lattice is duplicated in the direction of the lattice size 10, which we call the z direction. We use an antiperiodic boundary condition for quarks in the t direction and periodic boundary conditions otherwise.

We investigate confinement in the strong coupling limit $\beta = 0.0$ ($g = \infty$, $\beta = 6/g^2$). Although quark confinement is rigorously proved at $\beta = 0.0$ in the pure gauge theory when the action is local as in the case of the standard one-plaquette action [4], there is no proof for confinement in full QCD.

Let us begin with the case of $N_f = 18$, because asymptotic freedom is lost for $N_f \geq 17$ and therefore we may expect quark nonconfinement here. We take $T = 4$. Shown in Fig. 1(a) are the results of the Polyakov loop and the Wilson loop $W(1\times1)$ for various hopping parameters. The data with large symbols are for long runs: They are obtained by averaging over the last $\tau = 500$ [at each τ = integer here and in the following for $W(1\times1)$ and Polyakov loop] after a thermalization of $\tau = 300$–500. The others are for a cycle of changing hopping parameter: Starting from the last configuration of the long run at $K = 0.25$, the hopping parameter first decreases down to 0.17 and then the hopping parameter increases from 0.19 to 0.205 with about $\tau = 20$ at each hopping parameter. The data are taken over the last $\tau = 10$. The data of the long run show that the statistical fluctuation is very small and therefore the statistics of the short run is good enough for the thermalization and the data taking. Hence the statistics in the following is similar to that of the short run. The errors estimated by the jackknife method are smaller than the symbols in the figure here and in the following. We note that there are jumps of the Polyakov loop and the $W(1\times1)$ around $K = 0.20$. The magnitude of the jump is prominent for $W(1\times1)$. This

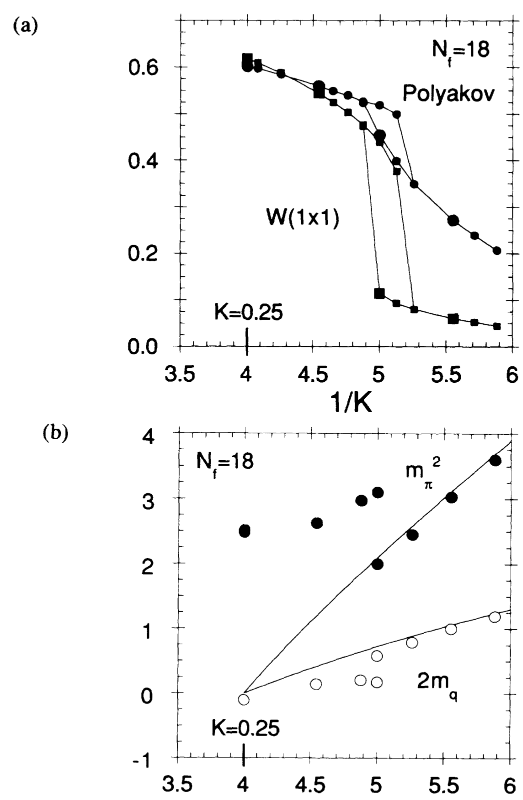

FIG. 1. Results for $N_f = 18$ at $\beta = 0.0$ on the $T = 4$ lattice with $\Delta\tau = 0.01$. (a) $W(1\times1)$ (squares) and Polyakov line (circles). The large symbols are for long runs. (b) m_π^2 (solid circles) and $2m_q$ (open circles).

result indicates that some kind of phase transition occurs around $K=0.2$.

To see more closely what happens, we calculate the hadron spectrum and the quark mass for various hopping parameters, typically on 20 configurations separated by $\tau=25$. The definition of the quark mass is identical with that in Ref. [2]. From $K=0.17$ to $K=0.2$, the pion mass m_π and the quark mass m_q are, as is shown in Fig. 1(b), in good agreement with the strong coupling calculations [1] without quark loops (the rho meson mass m_ρ, the nucleon mass m_N, and the delta mass m_Δ also agree with the corresponding mass formula):

$$\cosh m_\pi = 1 + \frac{(1-16K^2)(1-4K^2)}{4K^2(2-12K^2)}, \tag{1}$$

$$2m_q = m_\pi \frac{4K^2 \sinh m_\pi}{1-4K^2\cosh m_\pi}. \tag{2}$$

Both m_π and m_q given by these formulas vanish at $K=0.25$. Thus in this phase the quark is confined, and chiral symmetry is spontaneously broken in the sense that if we could take the chiral limit, m_π would vanish.

On the other hand, between $K=0.20$ and 0.25 the m_q values suddenly become very small and are almost zero, and the m_π's are larger than that at $K=0.19$. In this region where m_q is very small, chiral symmetry is almost exact and manifest as is seen in the degeneracy of π and δ, ρ and A_1, as well as baryons and their parity partners: At $K=0.25$, $m_\pi=1.584(2)$ and $m_\delta=1.64(2)$, $m_\rho=1.61(1)$ and $m_{A_1}=1.64(2)$, $m_N=2.81(6)$ and $m_{N^{(-)}}=3.0(3)$, and $m_\Delta=2.81(3)$ and $m_{\Delta^{(-)}}=2.8(2)$.

The mass of pion at $K=0.25$ is about 1.6: We have made two independent calculations with $\Delta\tau=0.01$ and 0.005 at $K=0.25$ and confirmed that the results for the quark mass and the pion mass are completely consistent with each other. The lowest Matsubara frequency for a state of two quarks is $2\pi/T$ ($=1.57$ for $T=4$). The pion mass is almost equal to this value or slightly greater than it. Therefore in the following we call this state and a similar state a quark deconfining state.

Thus we conclude that for $N_f=18$ quarks are not confined and chiral symmetry is not spontaneously broken for $K \geq 0.20$ at $\beta=0.0$ on the $T=4$ lattice. At $K=0.20$ there is a jump of m_π, which indicates that the deconfining transition is first order.

We interpret the above phenomena as follows: When a quark mass is heavy in the sense that it is much larger than the inverse of the lattice spacing, the effect of quark loops should be negligible and therefore the system should belong to the same universal class as the quenched QCD. Thus the quark should be confined. When the quark mass becomes smaller, the effect of quark loops becomes crucial and there is a possibility that the system enters into a new phase because of this effect. When N_f is as large as 18, this effect indeed triggers the transition observed above.

To investigate whether the transition is a finite temperature transition or a transition at zero temperature, we have to investigate the T dependence of the transition. This will be discussed later in the case of $N_f=7$.

Now, the issue is to what number of flavors the property observed for $N_f=18$ holds. To investigate this problem we gradually decrease the number of flavors from $N_f=18$ by 2 units at a time down to $N_f=8$ and then to $N_f=7$, fixing the hopping parameter at $K=0.25$. Starting from the last configuration at $K=0.25$ for $N_f=18$, we make simulations of about $\tau=20$ for each flavor, taking the last configuration as the starting configuration of the next flavor. With decreasing number of flavors, $W(1\times1)$ averaged over the last $\tau=10$ decreases and the number of iterations needed for quark matrix inversion (by CG) increases as shown in Fig. 2(a). However, up to this point the changes are smooth and nothing peculiar happens.

When we switch the number of flavors from 7 to 6, a sudden change occurs. The number of iterations for the quark matrix inversion gradually increases and exceeds 10000 at $\tau=9$. In parallel with this, $W(1\times1)$ gradually decreases down to 0.08 at $\tau=8$ [see Fig. 2(b)]. Thus the case $N_f=6$ is completely different from the cases $N_f=7$–18.

To see further the difference between $N_f=6$ and the other values, we calculate the eigenvalues of $\gamma_5 D$ for gauge configurations, by varying the hopping parameter in D (which we call the valence hopping parameter $K_{\rm val}$) around $K_{\rm val}=0.25$: For $N_f=18$, 12, 8, and 7, one configuration for each flavor is chosen after thermalization, and for $N_f=6$, one configuration at $\tau=8$, although this is not an equilibrium state as discussed above. We see a clear contrast between the $N_f=6$ case and the others: There are no zero eigenvalues around $K_{\rm val}=0.25$ for $N_f=7$, 8, 12, and 18 and the smallest absolute eigenvalue gradually decreases from $N_f=18$ to 7 as 0.23, 0.16, 0.07, 0.04. On the other hand, for $N_f=6$ there is a zero eigenvalue at $K_{\rm val}=0.258$, as is shown in Fig. 3 for the cases of $N_f=6$ and 7. Our previous works show that the existence of zero eigenvalues around K_c is common to the case where the quark is confined, in contrast to the finite temperature deconfining phase [2,5]. Because of this fact it is very difficult to invert the quark matrix at $K \sim K_c$ in the confining phase, while it is easy to do so in the

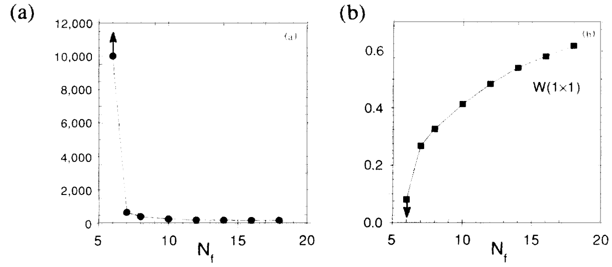

FIG. 2. Results for $N_f=18$–6. (a) Number of iterations needed for the quark matrix inversion by CG. (b) $W(1\times1)$.

deconfining phase: We will use this difference later as an indicator to discriminate the two phases.

The above results imply that at $K=0.25$ the phase for $N_f=16$–7 is identical to that of $N_f=18$ and is different from that of $N_f=6$. To strengthen this assertion we will investigate in more detail the cases of $N_f=7$ and 6 in the following.

The results of $W(1\times1)$ for $N_f=7$, together with those of the quark mass and the pion mass, on the $T=4$, 6, 8, and 18 lattices are given in Fig. 4: The data between $K=0.23$ and 0.24 for $T=4$ are with $\Delta\tau=0.005$ and the data at $K=0.23$ for $T=6$ are with $\Delta\tau=0.0025$, because we need smaller $\Delta\tau$ as the smallest absolute eigenvalue of the quark matrix D becomes smaller. $W(1\times1)$ for $T=4$ has a jump between $K=0.24$ and 0.245. The $W(1\times1)$ values for $T=18$ at $K=0.25$ and 0.245 are completely consistent with those for $T=4$. This implies that the transition is not a high-temperature phenomenon but a phenomenon at zero temperature, because if it were a finite temperature transition, the transition point would move towards a larger hopping parameter for $T=18$ and therefore $W(1\times1)$ at $K=0.245$ for $T=18$ would take a smaller value around 0.1. In accord with this behavior of $W(1\times1)$, m_π also shows a jump between $K=0.24$ and 0.245. From $K=0.20$ to 0.24, m_π is consistent with the strong coupling result without quark loops given by Eq. (1). The m_π values for $K\geq0.245$ are completely off the curve given by Eq. (1). The m_π's at $K=0.25$ are 1.251(5), 1.136(4), 1.137(5), and 1.121(2) for $T=4$, 6, 8, and 18, respectively. m_π decreases slightly with T. The m_π's for $T=6$, 8, and 18 are larger than the lowest Matsubara frequency. The Polyakov loop also has a jump between $K=0.24$ and 0.245. Therefore we interpret the state for $K\geq0.245$ as a deconfined state. Thus we conclude that when the quark mass is light, the quark is not confined and chiral symmetry is not spontaneously broken for $N_f=7$ at zero temperature. Although we believe that this confirmation is enough for the cases for $N_f=8$–18 because the $N_f=7$ case is the critical case, we have checked a similar thing for the $N_f=12$ case.

As mentioned already, at $K=0.25$ the $N_f=6$ case is completely different from the cases $N_f=18$–7. One strong possibility is that the point $K=0.25$ at $\beta=0.0$ for $N_f=6$ belongs to the confining phase. To see whether this is correct we investigate the problem [6] of whether the finite temperature transition line $K_t(\beta)$ on the $T=4$ lattice crosses the chiral limit line $K_c(\beta)$ at finite β, by monitoring the number of iterations of CG needed for the matrix inversion at several points on the line $K_c(\beta)$: $K=0.22$, 0.24, 0.245, 0.2475, and 0.2495 at $\beta=4.0$, 2.0,

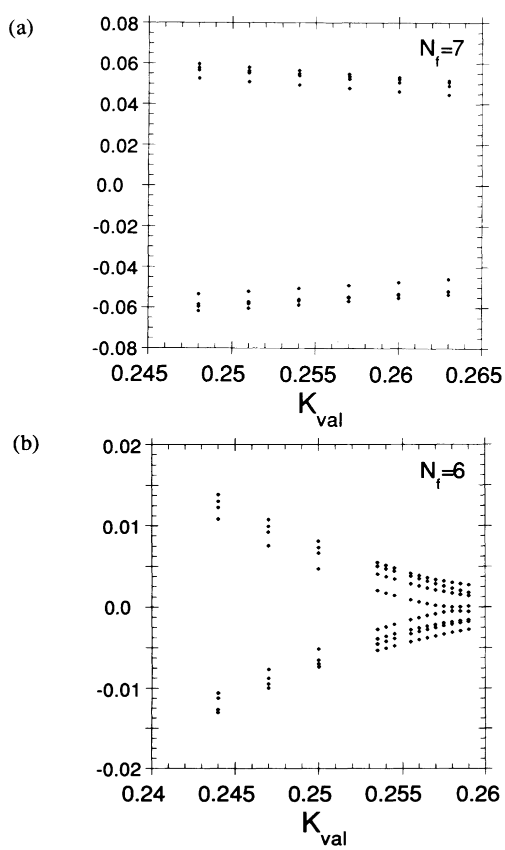

FIG. 3. Eigenvalues of $\gamma_5 D$ vs K_{val}. (a) $N_f=7$. (b) $N_f=6$.

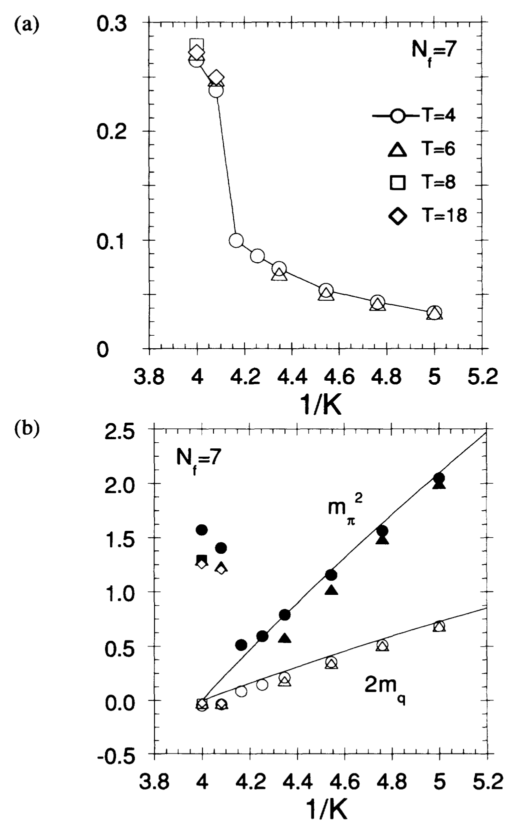

FIG. 4. Results for $N_f=7$ at $\beta=0.0$: Circles, $T=4$; triangles, $T=6$; squares, $T=8$; and diamonds, $T=18$. (a) $W(1\times1)$. (b) m_π^2 and $2m_q$.

1.0, 0.5, and 0.1, respectively [7]. As mentioned earlier, in the deconfining phase the number of iterations is moderate close to the critical line, while it is quite large in the confining phase. We find that the iteration numbers are 170–890 for $\beta=4.0$–0.5, while it exceeds 10000 at $\beta=0.1$. In accord with this behavior, $W(1\times1)$ decreases to 0.04 at $\tau=11$ for $\beta=0.1$, while the other values are larger than 0.2. The increase of the number of iterations together with the decrease of $W(1\times1)$ strongly implies that the line $K_t(\beta)$ crosses the line $K_c(\beta)$ between $\beta=0.1$ and 0.5. (For $N_f=2$ on the $T=4$ lattice, it crosses above $\beta=2.0$.) The work to strengthen this statement, by identifying the crossing point in more detail and by measuring physical quantities around the crossing point, is in progress. The details of this investigation will be given elsewhere.

The crossing of the line $K_t(\beta)$ and the line $K_c(\beta)$ at finite β implies confinement at smaller β for $K\leq K_c$. Therefore it leads to quark confinement for $K\leq K_c$ at $\beta=0.0$ in the case of $N_f=6$. To see whether what really happens is consistent with this observation, we make the calculation at $\beta=0.0$ for $N_f=6$. If a zero eigenvalue exists around $K=0.25$, we will be unable to calculate the physical quantities at $K=0.25$ by known algorithms. Here we are able to calculate physical quantities up to $K=0.24$ within reasonable CPU time: $K=0.20$, 0.21, 0.22, 0.235, and 0.24 ($\Delta\tau=0.005$ for $K=0.24$). The results for m_π and m_q are completely consistent with Eqs. (1) and (2) up to $K=0.24$. Thus, up to the point that we have been able to calculate, the results support the conclusions that the chiral limit is $K=0.25$ and that the point $K=0.25$ belongs to the confining phase.

Now we state the major conclusion of this Letter: For $N_f\geq7$ quarks are not confined and chiral symmetry is manifest for light quarks at $\beta=0.0$. (Although we have not investigated the case for $N_f>18$, we conjecture that these features hold for this case also.) Combining the above results with further analyses for $N_f=12$ and 8 that we have made, we can draw a map of the deconfining-transition hopping parameter K_d versus the number of flavors, as shown in Fig. 5.

The fundamental problem which remains is whether this property holds in the continuum limit. This is beyond the scope of this Letter. The numerical work to investigate this problem is in progress. We hope that we are able to report the result in the near future.

The dependence of chiral symmetry on the number of flavors in the Kogut-Susskind formalism was previously investigated [8]. There was no indication that for a large number of flavors chiral symmetry is recovered. However, after this work was completed, we became aware of a report [9] that there is an indication of a bulk transition for $N_f=8$. It is not clear whether the phenomenon reported is related to the transition observed by us. Further investigation to clarify this point is certainly necessary.

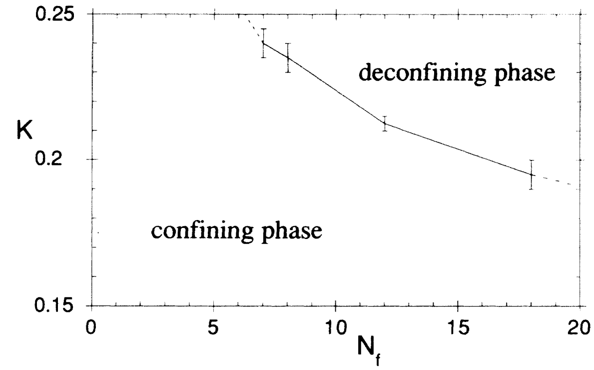

FIG. 5. Phase diagram at $\beta=0.0$: Deconfining-transition hopping parameter K_d vs the number of flavors. The line is to guide the eyes.

The numerical calculations on the $8^2\times10\times T$ ($T=4$, 6, or 8) lattices and the $18^2\times24\times T$ ($T=18$) lattice have been performed with HITAC S820/80 at KEK and with QCDPAX at University of Tsukuba, respectively. We would like to thank members of KEK for their warm hospitality and strong support and the other members of the QCDPAX collaboration for their help. We would also like to thank Sinya Aoki and Akira Ukawa for valuable discussions. This project is in part supported by a Grant-in-Aid of Ministry of Education, Science and Culture (No. 62060001 and No. 02402003).

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